A simple nonlinear filter for low-cost ground vehicle localization system

S. Bonnabel (Mines ParisTech)
Joint work with
J.E. Deschaud (Carnegie Mellon University)
E. Salaün (SAGEM)

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Introduction

Standard commercial GPS only have a relative accuracy (a couple of meters error). Aiding them with inertial sensors (IMU) can lead to substantial gains.

Recent technological developments have lead to a wide-spread use of low-cost accelerometers and gyrometers. The signals they deliver are biased, and the biases tend to slowly drift with temperature. They need to be checked against GPS which is precise in average.

Here we study a way to fuse those data for the specific example of a car in urban aera.

Introduction

- In urban canyons (or tunnels), the GPS signal is degraded.
- GPS can be aided by "cheap" inertial sensors (MEMS) and speedometer.
- Using a roll without slip model of the car can yield substantial gain[1] in the position estimation performance.

[1] e.g. Fiengo, Di Domenico, Glielmo. CEP. 2009.

Introduction

IMU/GPS data fusion is well-established via EKF.

Over the last years, alternative observers have been developed for fusion with low-cost sensors[2] :

- they take into account the nonlinear structure of the problem,
- they have guaranteed convergence properties,
- they are easy to implement.

In this talk, we consider application to a car with IMU, GPS and speedometer.

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[2]Bonnabel, Martin, Rouchon. Symmetry-preserving observers. IEEE-TAC 2008.
Mahony, Hamel, Pflimlin. IEEE-TAC 2008.
Vasconcelos, Cunha, Silvestre, Oliveira. SCL 2010.

Introduction

We propose a simple nonlinear observer that has the following advantages:

- **Convergence properties:** especially, the yaw angle (ψ) estimate converges almost globally.
- **Precision at low speed:** usually $\psi \simeq \arctan(\frac{V^n_{GPS}}{V^e_{GPS}})$, which has severe limitations.
- **Simplicity:** the observer is easy to tune (only a few parameters to choose), easy to implement and computationally economic (very few scalar operations).

Outline

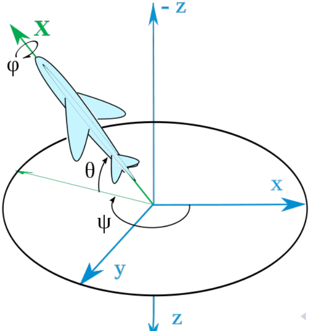
1. Ground vehicle localization problem
2. Nonlinear observer
3. Experiments in Paris

Sensors (inputs):

- gyrometers: biased angular velocity vector $\omega \in \mathbb{R}^3$
- accelerometers: kinematic acceleration minus gravity vector field in body frame: $a - g \in \mathbb{R}^3$
- speedometer: wheel speed (available from Anti-lock Braking System)

Measurements: GPS position $p \in \mathbb{R}^3$ and velocity $v \in \mathbb{R}^3$.

Estimation: position and orientation of the car.



Model: nonholonomic equations[3]

$$
\begin{aligned}
\dot{\phi} &= \omega_x - b_x + \tan(\theta)((\omega_y - b_y)\sin\phi + (\omega_z - b_z)\cos\phi) \\
\dot{\theta} &= (\omega_y - b_y)\cos\phi - (\omega_z - b_z)\sin\phi) \\
\dot{\psi} &= (\omega_z - b_z)\cos\phi + (\omega_y - b_y)\sin\phi)/\cos(\theta) \\
\dot{p} &= v_s(\cos(\theta - \theta_0)\cos\psi, \cos(\theta - \theta_0)\sin\psi, \sin(\theta - \theta_0))^T \\
\dot{b}_x &= \dot{b}_y = \dot{b}_z = \dot{\theta}_0 = 0,
\end{aligned}
$$

with inputs ω, v_s . GPS measurements yield (when available)

$$
\begin{pmatrix} y_p \\ y_v \end{pmatrix} = \begin{pmatrix} p \\ v \end{pmatrix},
$$

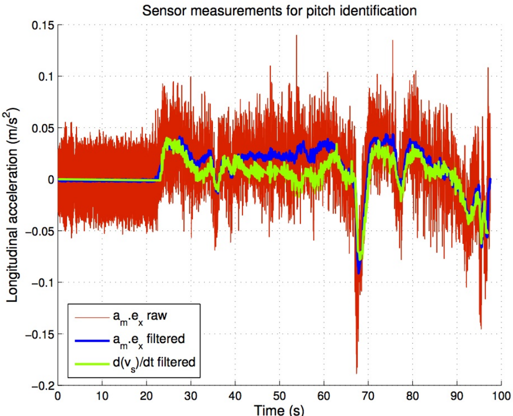
where $v = v_s(\cos(\theta - \theta_0)\cos\psi, \cos(\theta - \theta_0)\sin\psi, \sin(\theta - \theta_0))^T$.
Recall we do not want to divide by measurements.

[3] with biases +1 harmonization angle θ_0

Additional measurements

Accelerometers measure $a_m = a - g$ in the body frame. But the nonholonomic constraint implies $v = v_s e_x$.

We check experimentally that the longitudinal accelero gives the same information as the time derivative of the speedometer measurement when the car moves in horizontal straight line.



Additional measurements

The e_x component of the kinematic acceleration a can be removed computing the following time derivative $\dot{v}_s$. The centrifugal forces are estimated via ω with bias.

As $a_m = a - g$, the accelero signal can be transformed to obtain the following output (that yields pitch and roll):

$$y_a = \begin{pmatrix} y_{a_x} \\ y_{a_y} \\ y_{a_z} \end{pmatrix} = \begin{pmatrix} g \sin\theta \\ -v_s b_z + g \sin\phi \\ v_s b_z + g \cos\theta \sin\phi \end{pmatrix}.$$

Further simplifications

Linearization around the horizontal attitude: assuming $\sin\theta \simeq \theta$ and $\sin\phi \simeq \phi$ is relevant on human made roads (a 30% slope corresponds to a cosine of 0.97).

Linearized system:

$$
\begin{aligned}
\dot{\phi} &= (\omega_x - b_x) + \theta\omega_z \\
\dot{\theta} &= (\omega_y - b_y) - \phi\omega_z \\
\dot{\psi} &= (\omega_z - b_z) + \omega_y\phi \\
\dot{p} &= v_s(\cos\psi, \sin\psi, \theta + \theta_0)^T \\
\dot{b}_x = \dot{b}_y &= \dot{b}_z = \dot{\theta}_0 = 0.
\end{aligned}
$$

with output $y_p = p, y_v = v$, and $y_\theta = \theta, y_\phi = \phi$ from y_a.

Proposed nonlinear observer

Nonlinear observer

Roll, pitch, and vertical subsystem:[4] estimation of

$$\begin{aligned} \phi, \theta, b_x, b_y \quad & \text{with} \quad y_\theta, y_\phi \\ \theta_0 \quad & \text{with} \quad z_{GPS}. \end{aligned}$$

Heading nonlinear subsystem: estimation of

$$\psi, b_z, p^n, p^e \quad \text{with} \quad v^n_{GPS}, v^e_{GPS}, p^n_{GPS}, p^e_{GPS}.$$

[4] easy to prove convergence provided $\int_0^\infty v_s dt = \infty$.

Heading nonlinear subsystem

After pre-compensating some terms, we finally focus on

$$\begin{aligned}
\dot{\psi} &= \omega_z - b_z \\
\dot{b}_z &= 0 \\
(y_{v^e}, y_{v^n}) &= (v_s \cos\psi, v_s \sin\psi)
\end{aligned}$$

where we do NOT want to divide by the possibly vanishing terms $v_s, \cos\psi, \sin\psi$.

Inspiring from previous work[5] we can make an observer adapted to the structure of rotations in the plane.

[5] Bonnabel, Martin, Rouchon. Symmetry-preserving observers. IEEE-TAC 2008.

We define the rotation-invariant nonlinear output error

$$\cos(\hat{\psi})y_{v^e} - \sin(\hat{\psi})y_{v^n} = v_s \sin(\hat{\psi} - \psi)$$

and we propose the nonlinear observer

$$\frac{d}{dt}\hat{\psi} = \omega_z - \hat{b}_z - k_\psi v_s \sin(\tilde{\psi})$$
$$\frac{d}{dt}\hat{b}_z = -k_b v_s^2 \sin(\tilde{\psi})$$

which yields the autonomous error system

$$\frac{d}{dt}\tilde{\psi} = -\tilde{b}_z - k_\psi v_s \sin(\tilde{\psi})$$
$$\frac{d}{dt}\tilde{b}_z = -k_b v_s^2 \sin(\tilde{\psi})$$

where $\tilde{\psi} = \hat{\psi} - \psi,\ \tilde{b}_z = \hat{b}_z - b_z$.

Almost global convergence for constant speed

Proposition

Assume the speed v_s is constant over the time. Set $k_\psi, k_b, k_p > 0$. The nonlinear observer is such that:

- *the error $(\tilde{\psi}, \tilde{p}^n, \tilde{p}^e, \tilde{b}_z)$ is locally exponentially stable to 0;*
- *for almost any initial conditions, the error asymptotically converges to 0.*

Remark: this condition is often met in normal driving conditions.

The proof is due to the nice structure of the error system.

Local convergence around a large set

Proposition

Assume there exist three scalars $M_1, \underline{M}_2, \bar{M}_2 > 0$, such that the speed v_s satisfies

$$\frac{d}{dt} v_s < M_1 v_s^2, \quad \underline{M}_2 \leq v_s^2 \leq \bar{M}_2.$$

Take $k_b > 0$ and $k_\psi > M_1$. The nonlinear observer converges around any trajectory of the time-varying corresponding subsystem.

Indeed the linearized subsystem writes

$$\frac{d}{dt}\tilde{\psi} = -\tilde{b}_z - k_\psi v_s \tilde{\psi}$$
$$\frac{d}{dt}\tilde{b}_z = k_b v_s^2 \tilde{\psi}.$$

Gain tuning

Heading subsystem: the linearized subsystem is a second order linear system

$$\frac{d^2}{dt^2}\tilde{\psi} + k_\psi v_s \frac{d}{dt}\tilde{\psi} + k_b v_s^2 \tilde{\psi} = 0$$

Let $k_\psi = 2\xi\omega_0$ and $k_b = \omega_0^2$ with $\xi = \sqrt{2}/2$.

Remark: the convergence speed automatically adapts to v_s (thus to Signal-to-Noise Ratio).

Roll, pitch and vertical subsystem: only a few parameters to tune.

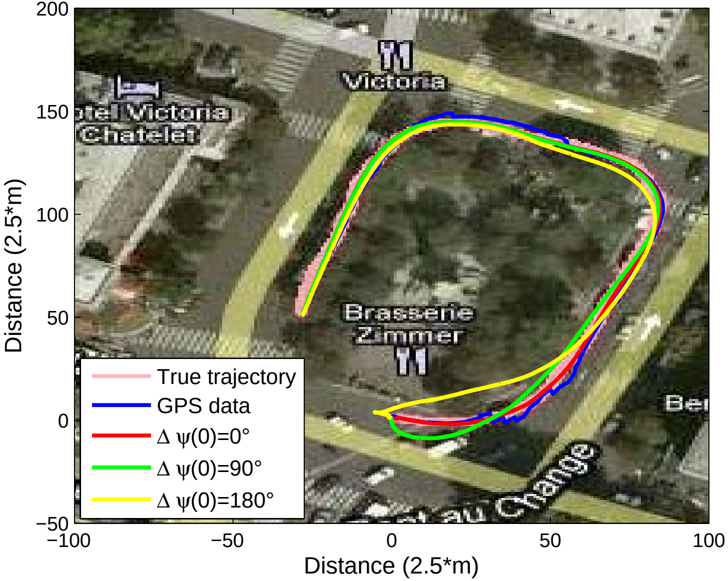
Experimental results

Experimental setup

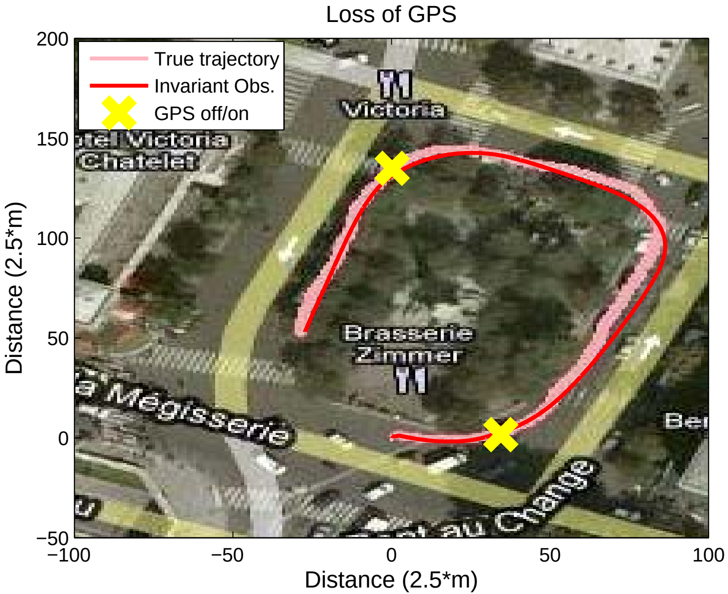
- Trajectory in the center of Paris
- Sensors
  - IMU, 84Hz
  - GPS, 10Hz
  - wheel speed sensors, 10Hz
- Implementation
  - Real-time, 84Hz
  - simple Euler explicit algorithm

Experiments: checking global convergence

Global behavior of the nonlinear observer



Experiments: simulating a GPS loss



Conclusion

Conclusion

We propose a nonlinear filter for ground vehicle localization that

- takes into account several sensor imperfections,
- is simple and easy to tune,
- possesses guaranteed convergence properties,
- provides online estimation of the slowly drifting gyro biases,
- is robust to GPS losses,
- well behaved at low-speed (useful for Mobile Mapping System),
- relies on simplifications, and can still be improved !